Polynomial Inequalities (1.5) HW

1. A polynomial p is given by $p(x) = (x^2 - 3x - 4)(x^2 - 8x + 16)$. What are all intervals on which $p(x) \geq 0$?

2. A polynomial p is given by $p(x) = (2x^2 + 3x - 5)(1 - x)$. What are all intervals on which $p(x) < 0$?

3. A polynomial p is given by $p(x) = -\frac{2}{3}(x+4)^2(x-1)^3(x)^2(x-2)$. Find all intervals on which $p(x) \leq 0$?

4. A polynomial p is given by $p(x) = (x-4)^2(x^2 - 2x + 5)$. What are all intervals on which $p(x) > 0$?

5. A polynomial p is given by $p(x) = 18x^3 - 45x^2 - 8x + 20$. What are all intervals on which $p(x) < 0$

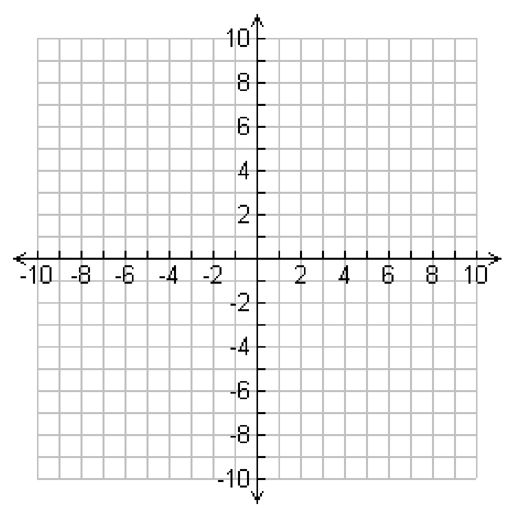
6. A polynomial p is given by $p(x) = x^4 + 5x^3 - 14x^2$. What are all intervals on which $p(x) \geq 0$?

7. Determine the domain of $f(x) = \sqrt{x^2(x-4)(x+1)^3}$

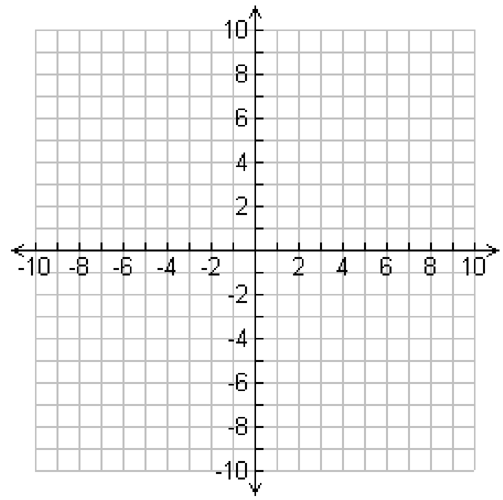
8. Determine the domain of $g(x) = \sqrt[4]{x(3-x)(5-x)^3}$

9. Determine the domain of $h(x) = \sqrt[3]{(x^2-9)(x^2+16)}$

10. On the graph provided to the right, sketch a polynomial function $f(x)$ for which the following are true:
    - The rate of change of $f(x)$ is always positive
    - The rate of change of $f(x)$ switches from decreasing to increasing at $x = -2$
    - $f(x) < 0$ on the interval $(-\infty, 1)$
    - $f(x) > 0$ on the interval $(1, \infty)$



11. On the graph provided to the right, sketch a polynomial function $f(x)$ for which the following are true:
    - $f(x) \geq 0$ for $x \in (-\infty, -2]$
    - $f(x) \leq 0$ for $x \in [-2, \infty)$
    - The graph of $f(x)$ is tangent to the x-axis at $x = 1$
    - On the interval $(-\infty, -1)$, $f(x)$ is decreasing and concave up
    - On the interval $(-1,0)$, $f(x)$ is increasing and concave up
    - On the interval $(0,1)$, $f(x)$ is increasing and concave down
    - On the interval $(1, \infty)$, $f(x)$ is decreasing and concave down



Each of the following questions describes a polynomial function. Decide if the scenario is possible or impossible. If impossible, give a reason why.

12. $f(x)$ is a fourth-degree polynomial, $f(x) < 0$ on $(-\infty, -3) \cup (3, \infty)$, $f(x) > 0$ on $(-3,3)$, $\lim_{x\to\infty} f(x) = -\infty$

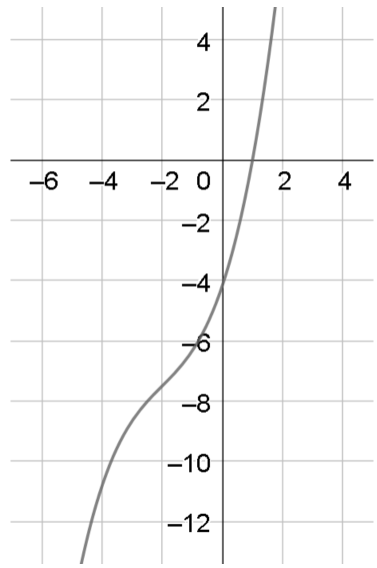
13. The leading coefficient of $f(x)$ is -4, $f(x)$ is cubic, $f(x) > 0$ on $(-2,0) \cup (2, \infty)$, $f(x) < 0$ on $(-\infty, -2) \cup (0,2)$

14. $f(x) \geq 0$ on $[2, \infty)$, $f(x) \leq 0$ on $[0,2]$, $f(x)$ is odd, $f(-3) = -10$

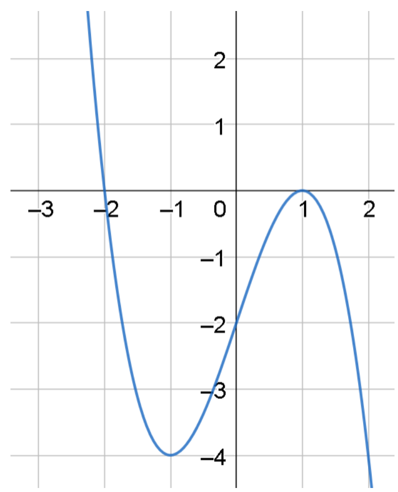
15. As the input of f decreases without bound the output increases without bound, $f(-2) = 1$ is a local maximum, there is a point of inflection at $x = 1$, $f(x) < 0$ on $(-\infty, -3)$.

Polynomial Inequalities (1.5) HW Answers

1. $(-\infty, -1] \cup [4, \infty)$
2. $\left(-\frac{5}{2}, 1\right) \cup (1, \infty)$
3. $(-\infty, 1] \cup [2, \infty)$
4. $(-\infty, 4) \cup (4, \infty)$
5. $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{2}{3}, \frac{5}{2}\right)$
6. $(-\infty, -7] \cup [0,0] \cup [2, \infty)$
7. $(-\infty, -1] \cup [0,0] \cup [4, \infty)$
8. $[0,3] \cup [5, \infty)$
9. $(-\infty, \infty)$ (It is a cube root function. Negative numbers can be the input to roots with an odd index. Even index roots are when the domain gets hard)



10. Answers may vary, but the graph should be increasing over its whole domain. It should be concave down for $x < -2$ and concave up for $x > -2$. The graph should be below the x-axis for $x < 1$ and above the x-axis for $x > 1$. One possible answer is to the right.



11. Answers may vary, but the basic shape should match the graph to the right. The graph should be above the x-axis for $x < -2$, should cross the x-axis at $x = -2$, and should be below the x-axis for $x = -2$ other than "bouncing" off the x-axis at $x = 1$. There should be a relative minimum at $x = -1$, a point of inflection at $x = 0$, and a relative maximum at $x = 1$.
12. Possible. In fact, the graph of $f(x) = 81 - x^4$ matches this description.
13. Impossible. A polynomial with a negative leading coefficient will have $\lim_{x\to\infty} f(x) = -\infty$. $f(x)$ is positive on the interval $(2, \infty)$, so it is impossible for the right end behavior to be $\lim_{x\to\infty} f(x) = -\infty$.
14. Possible. Given the symmetry of an odd function, knowing $f(x)$ is not negative for $x \geq 2$ leads us to also know that $f(x)$ is not positive for $x \leq -2$. This is consistent with $f(-3)$ being negative. No contradiction.
15. Impossible. In case it is a little too wordy, let's clarify that the statement "as the input of f decreases without bound the output increases without bound" is just saying $\lim_{x\to-\infty} f(x) = \infty$. This contradicts the given statement that $f(x)$ is negative on $(-\infty, -3)$.